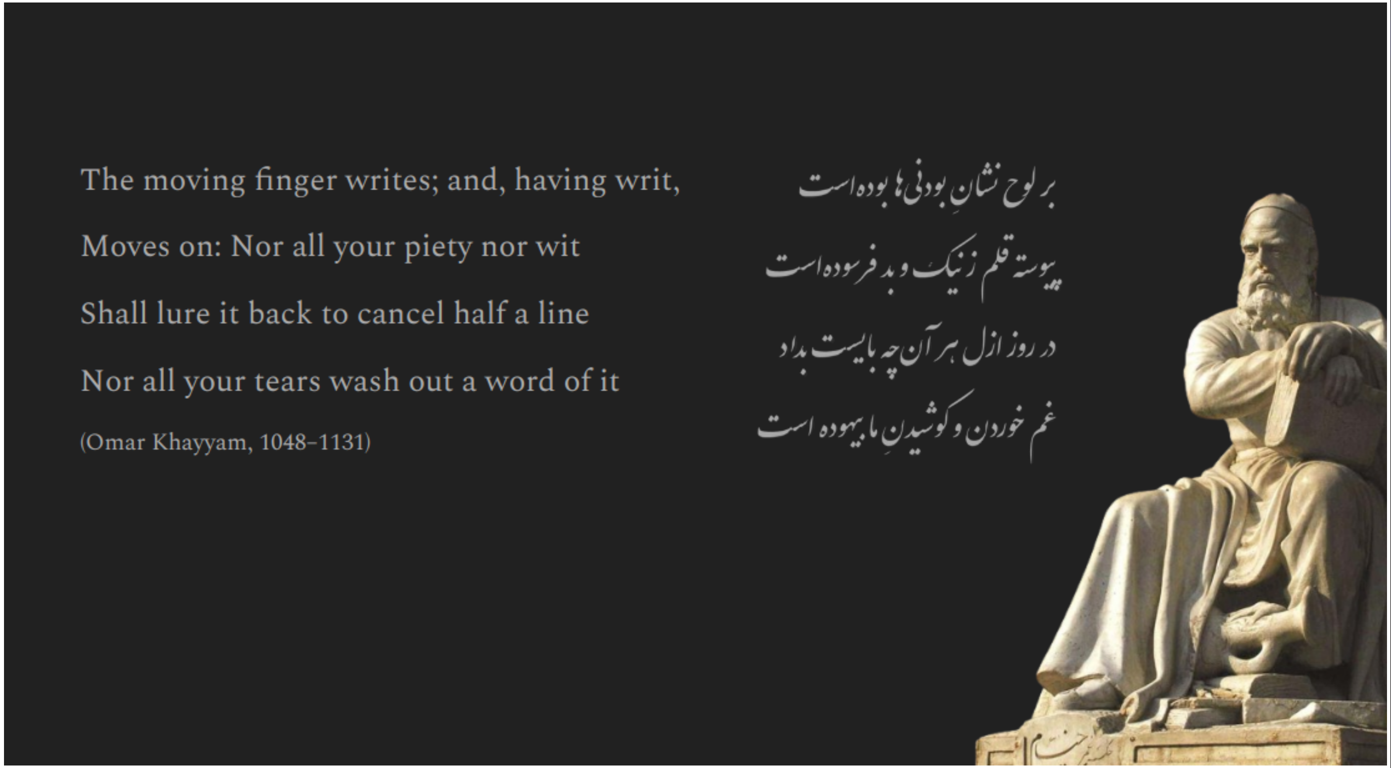
# My previous Talk was:

# I visited Omar Khayyam's tomb last month (:




## Today’s Talk

# Generative modeling,
# Diffusion Models,
# and Dynamics

# Dynamics

The subject that deals with change, with systems that evolve in time.

Whether the system:

1. Settles down to equilibrium
2. Keeps repeating in cycles
3. Does more complicated things like Chaos

# Dynamical systems:

1. Iterated Maps

example

**Mandelbrot fractal**

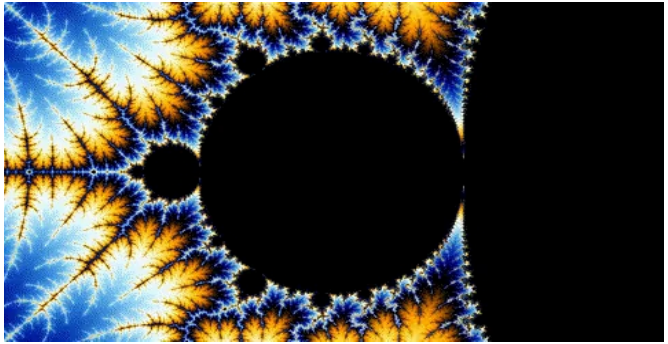
Set of hyperparameter c for which this iterated function:

$$z_{i+1} = z_i^2 + c$$

diverges or remains bounded (for an initial z value of 0);

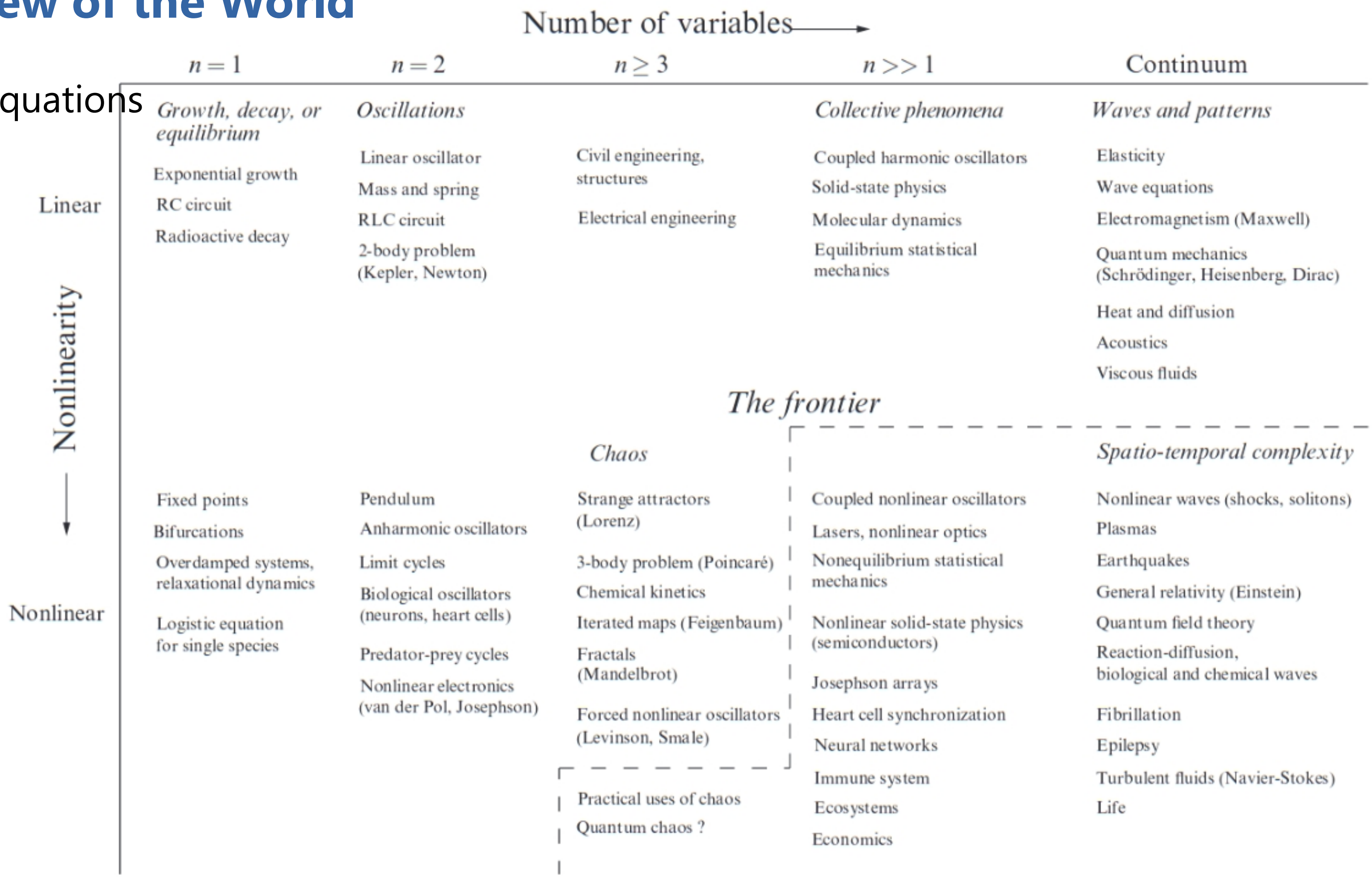
surprisingly complex behavior arises from a very simple mathematical relationship

It is self-similar under magnification in specific regions



# Dynamical View of the World

1. Differential equations

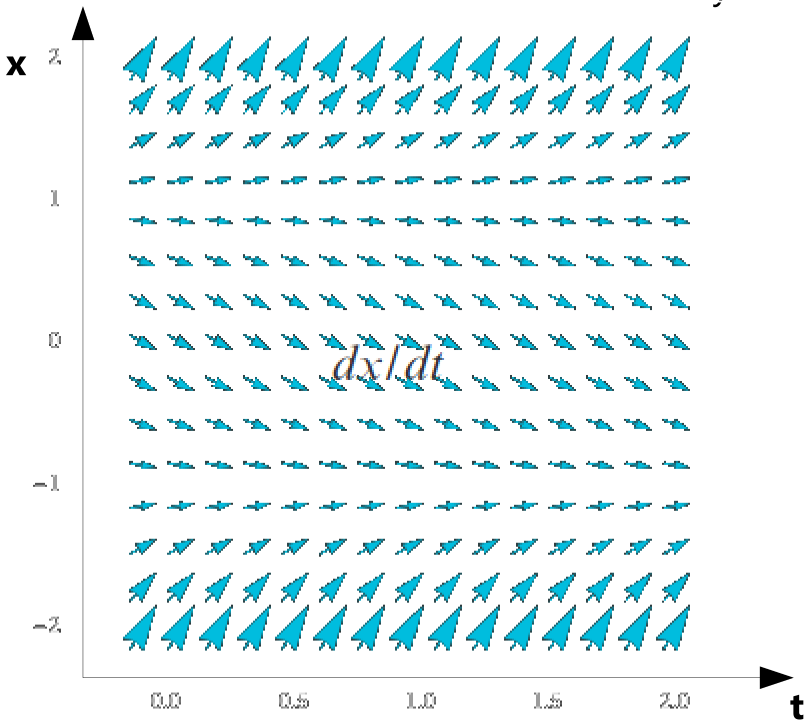


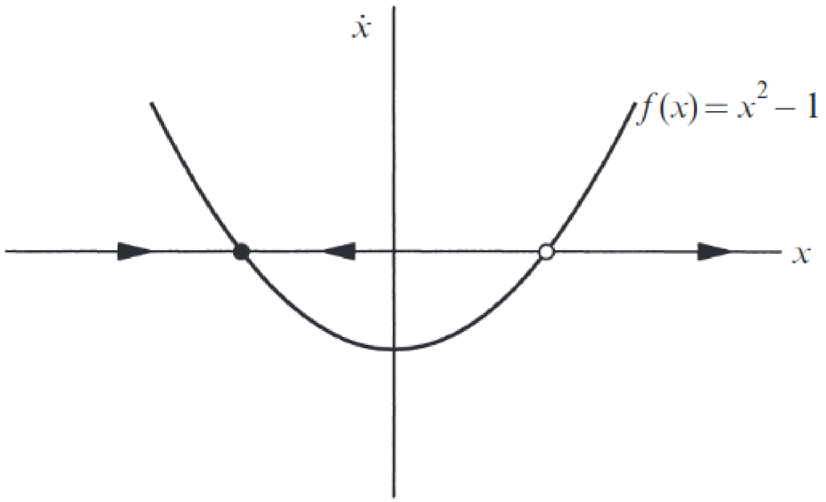

{Ref: Strogatz, S. H. (2019). Nonlinear dynamics and chaos (2nd ed.). London, England: CRC Press.}

# Stability of dynamical systems

$$dx/dt = x^2 - 1$$

This system has a stable point at x=-1 and an unstable point at x=1

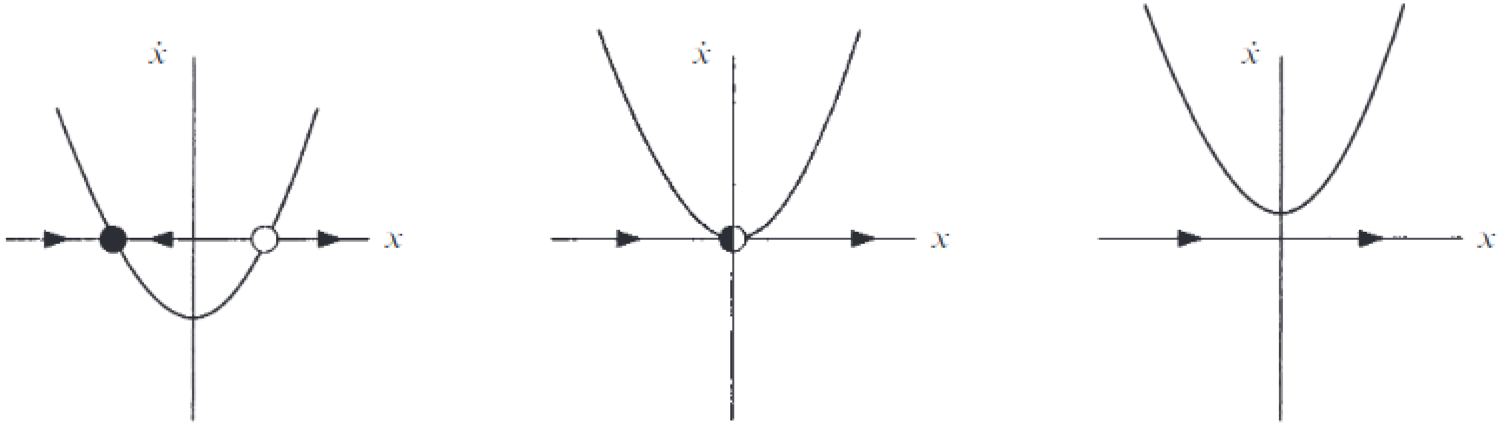


# BIFURCATIONS

qualitative changes in the dynamics

$$dx/dt = r + x^2$$




(a) $r < 0$ (b) $r = 0$ (c) $r > 0$

a bifurcation occurred at r = 0

# Spontaneous Symmetry Breaking in Generative Diffusion Models

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Poster, 37th Conference on Neural Information Processing Systems (NeurIPS 2023).

# Takeaways

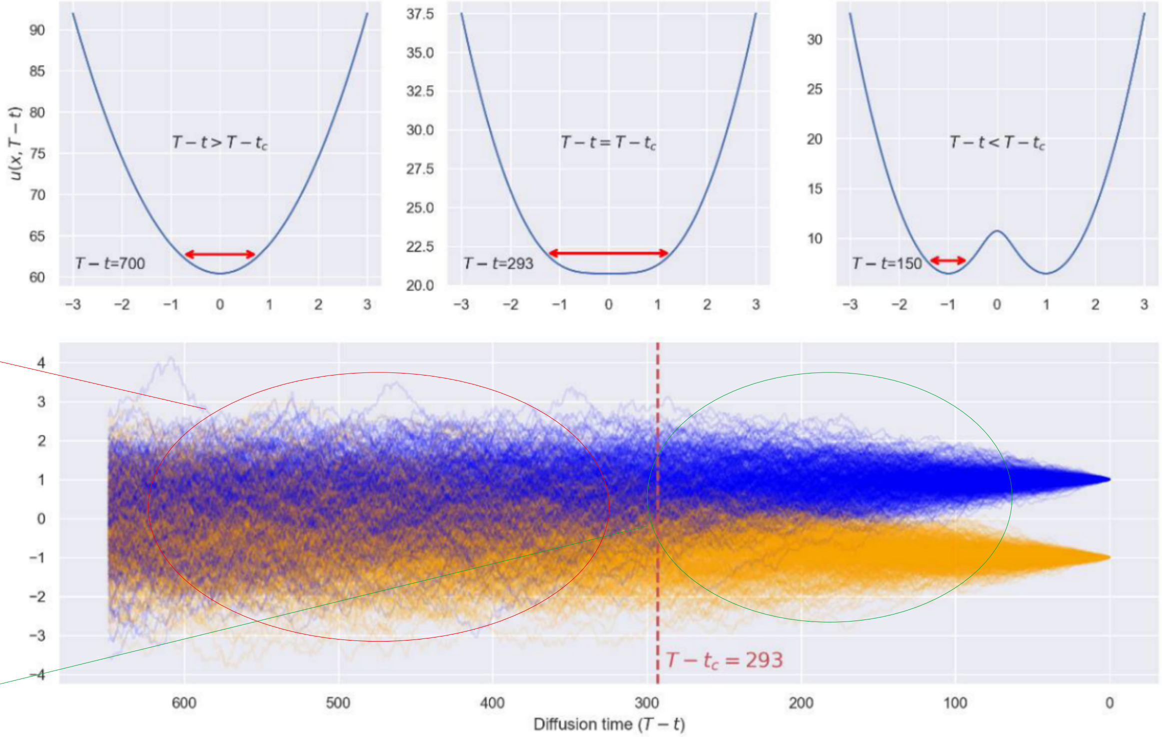
- Dynamics of diffusion models exhibit a spontaneous symmetry breaking.
- Early dynamics of sampling does not significantly contribute to the final generation,

# Spontaneous symmetry breaking in generative diffusion models.

one-dimensional problem
with two data points

A linear steady-state dynamics
around a central fixed-point

an attractor dynamics directed
towards the data manifold.

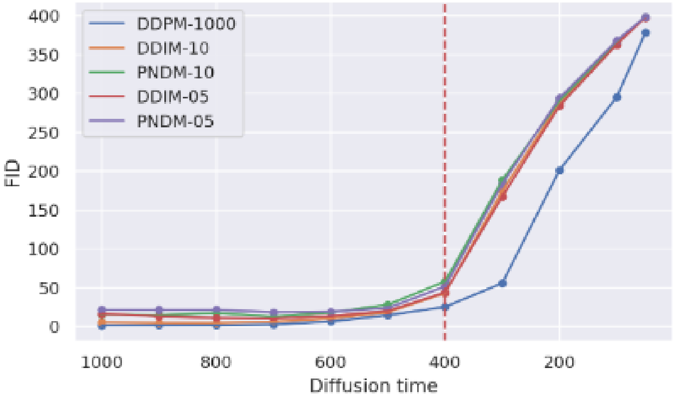



(a) Symmetry breaking in 1D diffusion model

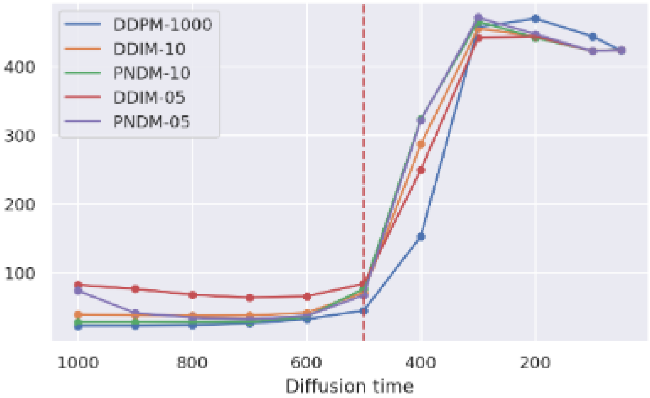
Figure 1: **Overview of spontaneous symmetry breaking in generative diffusion models.** a) Symmetry breaking in a simple one-dimensional problem with two data points (-1,1). The figures on the top illustrate the potential at different time points, while the bottom figure displays the stochastic trajectories. The red dashed line denotes the time of the spontaneous symmetry breaking (computed analytically). The red arrows represent fluctuations around the fixed-point of the drift.

# Impact of a late start initialization on FID performance

"in all datasets we see a sharp degradation in performance after
a certain initialization time threshold"

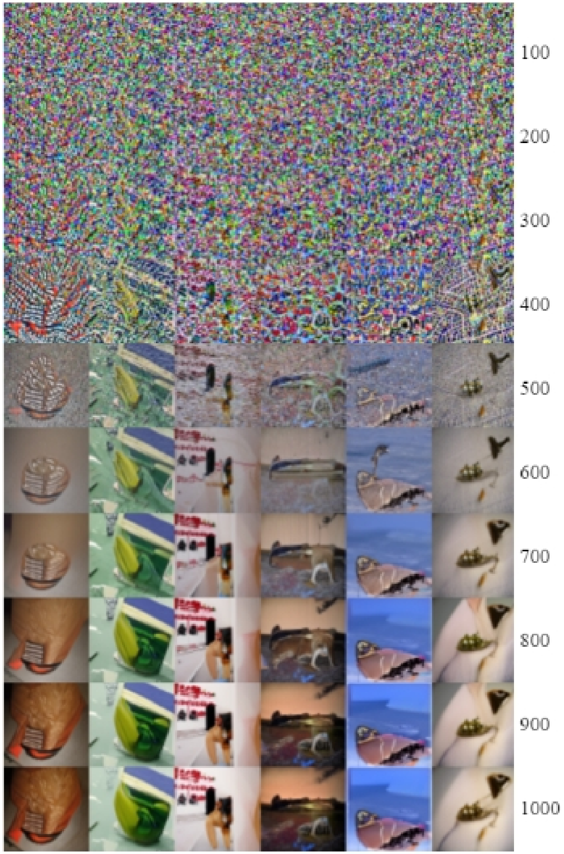



(a) MNIST




(c) Imagenet64

Figure 4: Analysis of the model's performance, as measured by FID scores, for different starting times using three different sampling methods: the normal DDPM sampler with decreasing time steps from $T = 1000$ to 0, and fast sampler DDIM and PSDM for 10 and 5 denoising steps. The vertical line corresponds to the maximum of the second derivative of the FID curve, which offers a rough estimate of the first bifurcation time. (e) Illustrates samples generation on Imagenet64, while progressively varying the starting time from 1000 to 100.




(e) Imagenet late start generation

Thank you for your attention!